Fraunhofer IIS

Fraunhofer-Institut für Integrierte Schaltungen IIS

Reinforcement Learning

Exercise 1: MDPs & Dynamic Programming

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Opening Remarks

Hello There!

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- Will take turns in holding the exercise session
- For specific question, please write us an E-Mail or (better) ask the question inside the StudOn forum
  - Will try to answer your questions together with Christopher Mutschler there
- You will find exercise sheet and code skeleton here: https://cmutschler.de/rl

Overview

Exercise Content

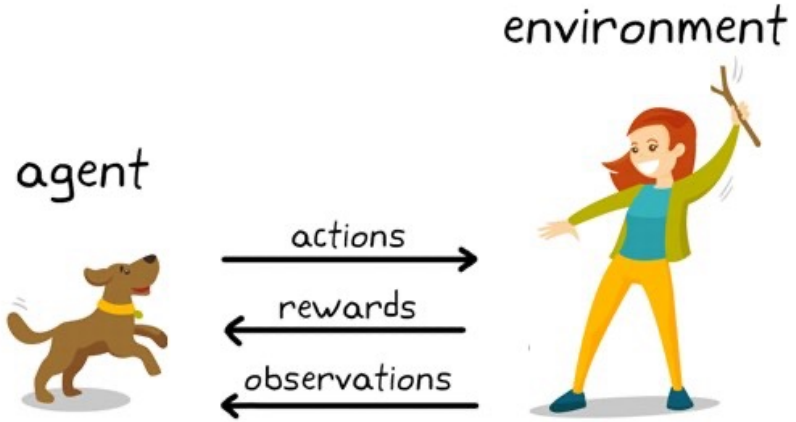
Week	*Date*	*Topic*	*Due Date (discussion of solution on...)*
1			
2	28.04.	MDPs	28.04.
	28.04.	Dynamic Programming	05.05.
3	05.05.	OpenAI Gym, TD-Learning	12.05.
4	12.05.	TD-Control	19.05.
5	19.05.	PyTorch, DQNs	02.06.
6	26.05.		
7	02.06.	VPG, A2C, PPO	16.06.
8	09.06.		
9	16.06.	MCTS	23.06.
10	23.06.	MPC, CEM	30.06.
11	30.06.	Multi-armed Bandits	07.07.
12	07.07.	RND/ICM	14.07.
13	14.07.	BCQ	19.07. (lecture slot)
14			

Basics: Reinforcement Learning

Intro to Reinforcement Learning

The RL paradigm

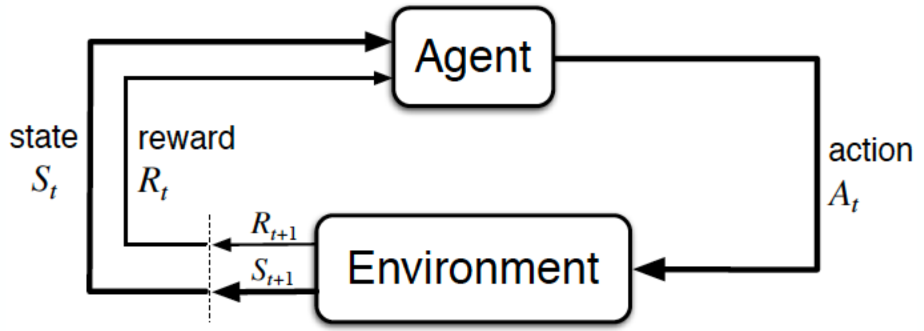
"All goals can be described by the maximization of expected cumulative reward."



Markov Decision Processes

Recap

- Agent learns by interacting with an environment over many time-steps:
- Markov Decision Process (MDP) is a tool to formulate RL problems
  - Description of an MDP $(\mathcal{S}, \mathcal{A}, \mathcal{P}, \mathcal{R}, \gamma)$



Sutton, R. S., & Barto, A. G. (2018). Reinforcement learning: An introduction. MIT press.

At each step t, the agent:
- is in a state S_t
- performs action A_t
- receives reward R_t

At each step t, the environment:
- receives action A_t from the agent
- provides reward R_t
- moves to state S_{t+1} with probability $P_{s,a}(s_{t+1})$
- increments time $t \leftarrow t + 1$

γ is the so-called discount factor

- If the interaction does stop at some point in time, then we have an *episodic* RL problem

Markov Decision Processes

Recap

- We need a controller that helps us select the actions to maximize expected cumulative reward
  - So-called: **Expected return** or **value**

- A policy π represents this controller:
  - π determines the agent's behavior, i.e., its way of acting
  - π is a mapping from state space $\mathcal{S}$ to action space $\mathcal{A}$, i.e., $\pi : \mathcal{S} \mapsto \mathcal{A}$
  - Two types of policies:
    - Deterministic policy: $a = \pi(s)$.
    - Stochastic policy: $\pi(a \mid s) = \mathbb{P}[A_t = a \mid S_t = s]$

- **Goal**: find a policy that maximizes the expected return!
  - We denote the optimal policy π for a given MDP as π^*

Markov Decision Processes

The Value Function

(State-)Value function

$$V_\pi(s) = \mathbb{E}_\pi[G_t \mid s_t = s\,] = \mathbb{E}_\pi\left[\sum_{t=0}^{\infty} \gamma^t r_t \mid s_t = s\right]$$

- "Expected return following policy π from state s"

Action-value function/Q-function

$$Q_\pi(s,a) = \mathbb{E}_\pi[G_t \mid s_t = s, a_t = a] = \mathbb{E}_\pi\left[\sum_{t=0}^{\infty} \gamma^t r_t \mid s_t = s, a_t = a\right]$$

- "Expected return of doing action a in state s and following policy π afterwards"

Exercise Sheet 1

Discussion

Dynamic Programming

Dynamic Programming
Introduction

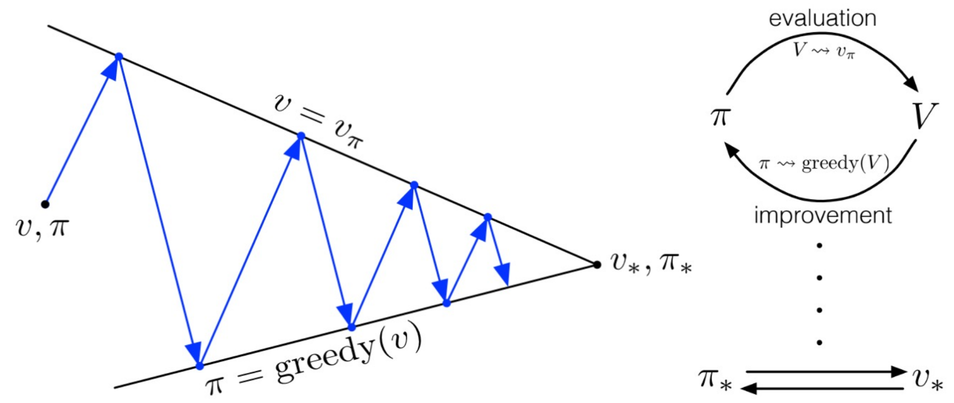
- Limited utility in practical reinforcement learning, but theoretical importance
  - Why?
- **Idea:** Use value functions to organize and structure the search for good policies
  - We can easily obtain optimal policies once we have found the optimal value function (and vice versa)
  - Founded on the Bellman optimality equation(s)

Bellman-Optimality Equation

$$V_{\pi^*}(s) = \max_a Q_{\pi^*}(s,a) = \max_a \mathbb{E}_{\pi^*}[r_t + \gamma V_{\pi^*}(s)]$$

Four key concepts

- Policy Evaluation
- Policy Improvement
- (Generalized) Policy Iteration
- Value Iteration



Sutton, R. S., & Barto, A. G. (2018). Reinforcement learning: An introduction. MIT press.

Policy Evaluation

- Given a policy π and the environment dynamics, we can easily compute the value of state:

$$V_\pi(s) = \mathbb{E}_\pi[G_t \mid s_t = s\,] = \;\dots = \sum_a \pi(a|s) \sum_{s'\,r} P(s',r \mid s,a)[r + \gamma V_\pi(s')]$$

- System of $\#states$ linear equations with $\#states$ unknowns
  - Can be solved straightforwardly
  - For our purposes, we solve it iteratively

Iterative Policy Evaluation, for estimating $V \approx v_\pi$

Input π, the policy to be evaluated
Algorithm parameter: a small threshold $\theta > 0$ determining accuracy of estimation
Initialize $V(s)$ arbitrarily, for $s \in \mathcal{S}$, and $V(terminal)$ to 0

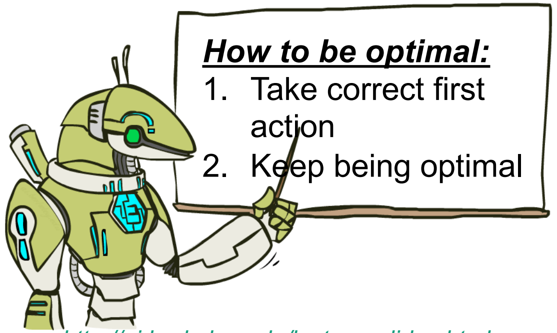
Loop:
  $\Delta \leftarrow 0$
  Loop for each $s \in \mathcal{S}$:
    $v \leftarrow V(s)$
    $V(s) \leftarrow \sum_a \pi(a|s) \sum_{s',r} p(s',r|s,a)\big[r + \gamma V(s')\big]$
    $\Delta \leftarrow \max(\Delta, |v - V(s)|)$
until $\Delta < \theta$

Policy Improvement

- Given a policy π and its value function (and the environement dynamics), greedily take the action that looks good in the short term

$$\pi'(s) = arg \max_a Q_\pi (s, a)$$

- Suppose $\pi' = \pi$, then π' fullfils the Bellman optimality equation in all states
  - Therefore: We found the optimal policy



http://ai.berkeley.edu/lecture_slides.html

Policy Iteration

Policy Iteration (using iterative policy evaluation) for estimating $\pi \approx \pi_*$

1. Initialization
   $V(s) \in \mathbb{R}$ and $\pi(s) \in \mathcal{A}(s)$ arbitrarily for all $s \in \mathcal{S}$; $V(\mathit{terminal}) \doteq 0$

2. Policy Evaluation
   Loop:
   $\quad \Delta \leftarrow 0$
   $\quad$ Loop for each $s \in \mathcal{S}$:
   $\quad\quad v \leftarrow V(s)$
   $\quad\quad V(s) \leftarrow \sum_{s',r} p(s',r|s,\pi(s))\big[r + \gamma V(s')\big]$
   $\quad\quad \Delta \leftarrow \max(\Delta, |v - V(s)|)$
   until $\Delta < \theta$ (a small positive number determining the accuracy of estimation)

3. Policy Improvement
   policy-stable $\leftarrow$ *true*
   For each $s \in \mathcal{S}$:
   $\quad$ *old-action* $\leftarrow \pi(s)$
   $\quad \pi(s) \leftarrow \arg\max_a \sum_{s',r} p(s',r|s,a)\big[r + \gamma V(s')\big]$
   $\quad$ If *old-action* $\neq \pi(s)$, then *policy-stable* $\leftarrow$ *false*
   If *policy-stable*, then stop and return $V \approx v_*$ and $\pi \approx \pi_*$; else go to 2

Value Iteration

- **Drawback of Policy Iteration:** We must do a full Policy Evaluation procedure for every step, which is costly!
- We can also truncate this:
  - If we stop the policy evaluation after just one sweep, this is called **Value Iteration**
  - Surprisingly, this corresponds to translating the Bellman optimality equation into an update rule
  - We can also drop the policy improvement step because we are only interested in the final policy

Iterative Policy Evaluation, for estimating $V \approx v_\pi$

Input π, the policy to be evaluated
Algorithm parameter: a small threshold $\theta > 0$ determining accuracy of estimation
Initialize $V(s)$ arbitrarily, for $s \in \mathcal{S}$, and $V(\mathit{terminal})$ to 0

Loop:
    $\Delta \leftarrow 0$
    Loop for each $s \in \mathcal{S}$:
        $v \leftarrow V(s)$
        $V(s) \leftarrow \sum_a \pi(a|s) \sum_{s',r} p(s', r \,|\, s, a)\big[r + \gamma V(s')\big]$
        $\Delta \leftarrow \max(\Delta, |v - V(s)|)$
until $\Delta < \theta$

Dynamic Programming
Summary

- Policy Evaluation
  - Given policy π, compute its (approximate) value function for (part of) the state space
- Policy Improvement
  - Given value function $V_\pi(s)$, extract the greedy policy $\pi'$with $V_{\pi'}(s) \geq V_\pi(s)$
- (Generalized) Policy Iteration
  - Repeat until convergence (policy doesn't change after improvement, i.e., Bellman optimality equation holds)
    - Do x steps of Policy Evaluation
    - Do Policy Improvement
- Value Iteration
  - Special case of Policy Iteration with 1 Policy Evaluation step
  - Converged when change in value estimates smaller than some threshold
  - Policy Improvement step only as the last step

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Thank you for your attention!